Lecture 12

Material and Homework

Read:

Chapter 16 Yariv and Yeh

Chapter 10 Yariv and Yeh

Chapter 11 Yariv and Yeh

Quantum Well Wavefunctions

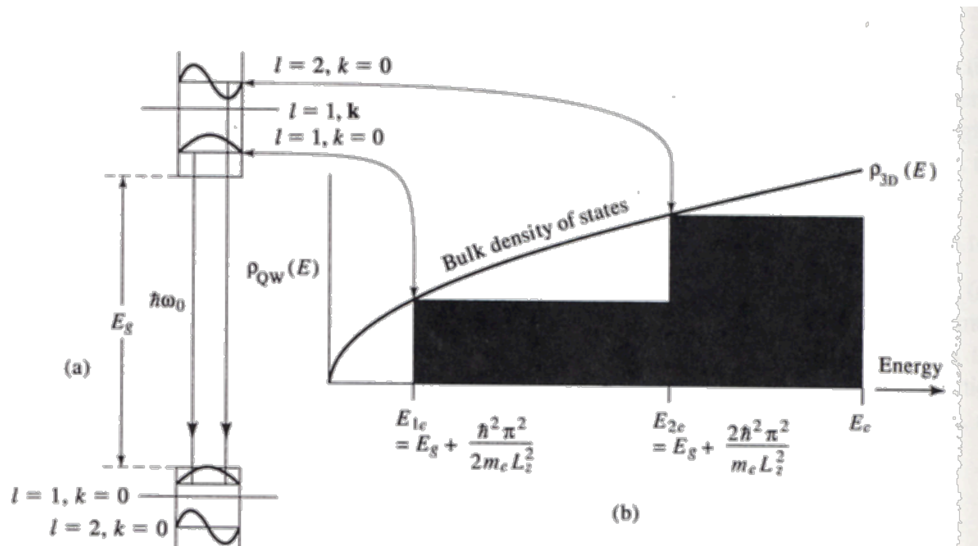
• The complete wavefunctions for electrons in the conduction band and holes in the valance band are given by, and are shown in the figure below.

$$\psi_c(r) = \sqrt{\frac{2}{L_z}}\psi_{k\perp c}(r_\perp)CS\left(l\frac{\pi}{L_z}z\right)$$

$$\psi_v(r) = \sqrt{\frac{2}{L_z}}\psi_{k\perp v}(r_\perp)CS\left(l\frac{\pi}{L_z}z\right)$$

$$\psi_c(r)_{groundstate} = \sqrt{\frac{2}{L_z}}\psi_{k\perp c}(r_\perp)Cos\left(\frac{\pi}{L_z}z\right)$$

$$\psi_v(r)_{groundstate} = \sqrt{\frac{2}{L_z}}\psi_{k\perp v}(r_\perp)Cos\left(\frac{\pi}{L_z}z\right)$$



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Density of States

• Assuming the electrons are confined to rectangles with L_x and L_y much larger than L_z, the K vectors are quantized according to

$$k_x = n\frac{\pi}{L_x}, n = 1,2,\ldots$$

$$k_y = m\frac{\pi}{L_x}, m = 1,2,\ldots$$

• The area in k space per eigenstate is $A_{state}=\pi^{2/} L_x L_y$, and the number of states as a function of k is $N(k)=k^2 L_x L_y/2\pi$

• The νumber of states between k and k+dk and the number of states with energies between E and E+dE are given by

$$\rho(k)dk = \frac{dN(k)}{dk} = L_x L_y \frac{k}{\pi} dk$$

$$\frac{dN(E)}{dE} dE = \frac{dN(k)}{dk}\frac{dk}{dE} = L_x L_y \frac{k}{\pi} dk$$

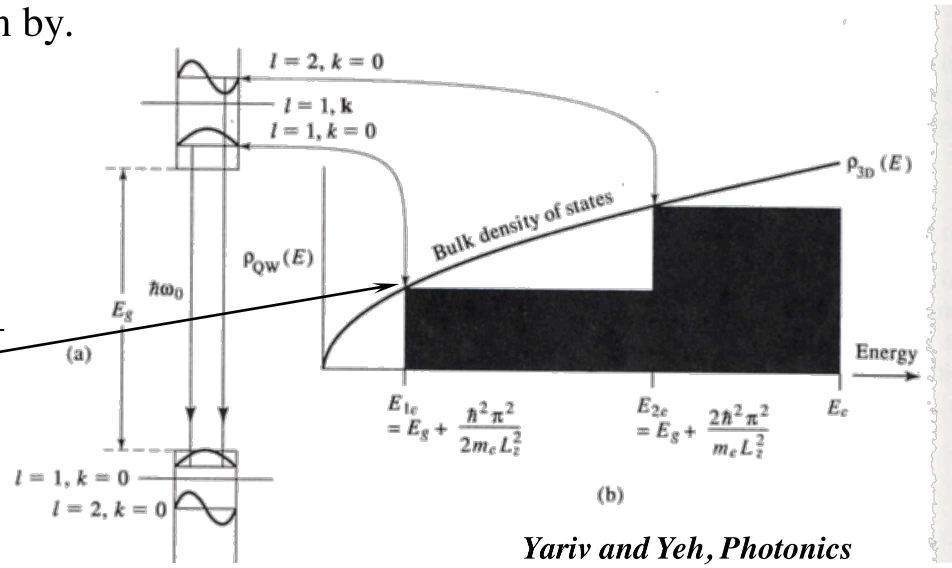
QW Gain Medium Density of States

• If we consider the lowest energy state in the conduction band, where electrons are free to move in 2D and confined in the third dimension by the well, the density of states per unit energy per unit area is constant and is given by.

$$\rho_{QW}(E) = \frac{1}{A_{\perp}}\frac{dN(E)}{dE} = \frac{k}{\pi}\frac{dk}{dE}$$

$$= \frac{1}{\pi}\sqrt{\frac{2m_c}{\hbar^2}}\left(E-E_{1c}\right)^{1/2}\frac{d\left(\sqrt{\frac{2m_c}{\hbar^2}}\left(E-E_{1c}\right)^{1/2}\right)}{dE}$$

$$= \frac{m_c}{\pi\hbar^2}$$



Yariv and Yeh, Photonics

• If the well can support the two lowest energy states (E_{1c} and E_{2c}) then the density of states doubles, and if it supports (E_{1c}, E_{2c} and E_{3c}) the density of states triples and so on leading to a staircase function shown above and given by (where H(x) is the Heaviside function = 1, x>0 and = 0, x<0)

$$\rho_{QW}(E) = \sum_{n=1}^{\text{all states}} \frac{m_c}{\pi\hbar^2} H(E-E_{nc})$$

Gain in QW Lasers

• The highest probability of stimulated recombination of and e-h pair via a photon is from l=1 in the conduction band to l=1 in the valence band. Therefore the highest optical gain will occur between these two levels.

• The l=1 electron state in the conduction band and l=1 hole state in the valence band must have the same l and k values, therefore, the transition energy can be written as

$$\begin{aligned}\hbar\omega &= E_c - E_v \\ &= E_g + E_c(k,l) + E_v(k,l) \\ &= E_g + \left(\frac{1}{m_c} + \frac{1}{m_v}\right)\frac{\hbar^2}{2}\left(k^2 + l^2\frac{\pi^2}{L_z^2}\right) \\ &= E_g + \frac{\hbar^2}{2m_r}\left(k^2 + l^2\frac{\pi^2}{L_z^2}\right)\end{aligned}$$

Gain in QW Lasers

• Using the quantum well carrier density $\rho_{QW}(k)/L_z$ the effective inverted population density due to carriers between k and $k+dk$ is given by

$$N_2 - N_1 \to \frac{kdk}{\pi L_z}\left[f_c(E_c) - f_v(E_v)\right]$$

• The gain due to electrons within dk and a single sub-band (e.g. l=1) can be written as a function of w assuming T_2 is the coherence collision time of an electron and τ the electron-hole recombination lifetime

$$\gamma(\omega_0)\Big|_{l=1} = \frac{m_r \lambda_0^2}{4\pi\hbar L_z n^2 \tau}\int_0^\infty \left[f_c(\hbar\omega) - f_v(\hbar\omega)\right]\frac{T_2}{\pi\left[1+(\omega-\omega_0)^2 T_2^2\right]}$$

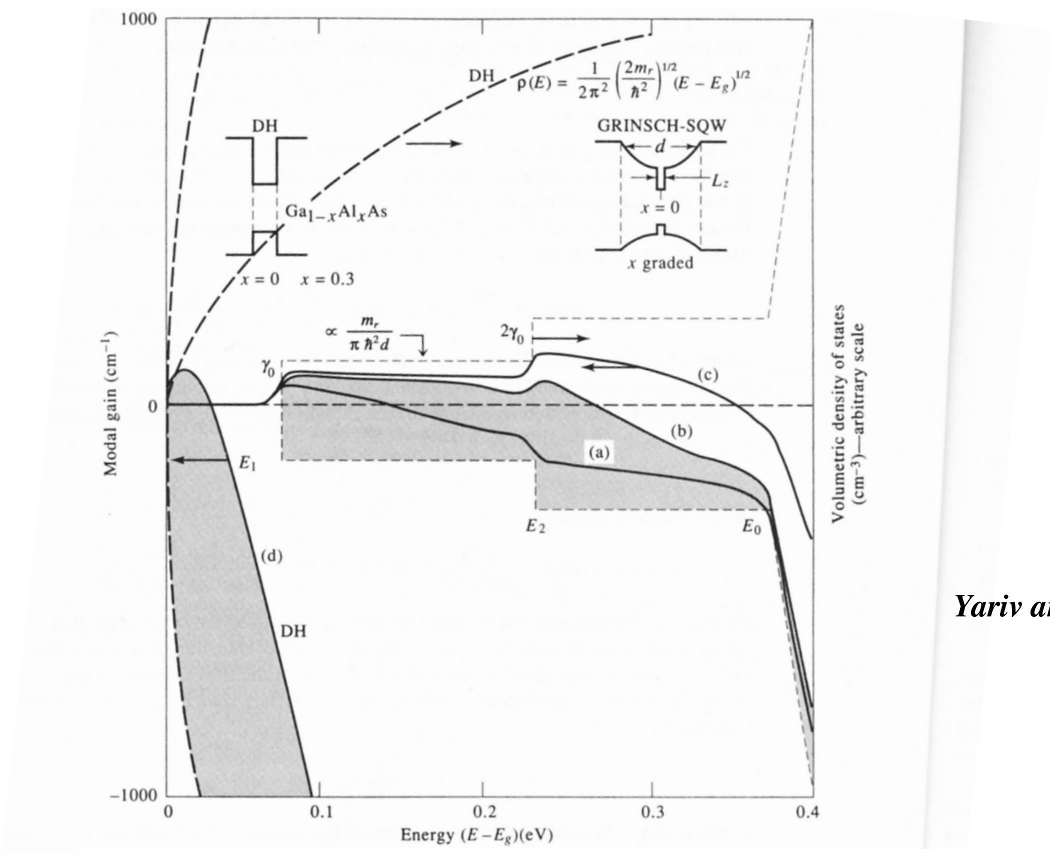
• Including the contributions from all sub-bands we assume

$$\frac{m_r}{\pi\hbar^2} \to \frac{m_r}{\pi\hbar^2}\sum_{l=1}^{\infty} H(\omega - \omega_l)$$

$$\frac{T_2}{\pi\left[1+(\omega-\omega_0)^2 T_2^2\right]} \to \delta(\omega - \omega_0)$$

$$\gamma(\omega_0) = \frac{m_r \lambda_0^2}{4\pi\hbar L_z n^2 \tau}\left[f_c(\hbar\omega_0) - f_v(\hbar\omega_0)\right]\sum_{l=1}^{\infty} H(\hbar\omega_0 - \hbar\omega_l)$$

Gain in QW Lasers



Yariv and Yeh, Photonics

Pulsed Lasers: Motivation

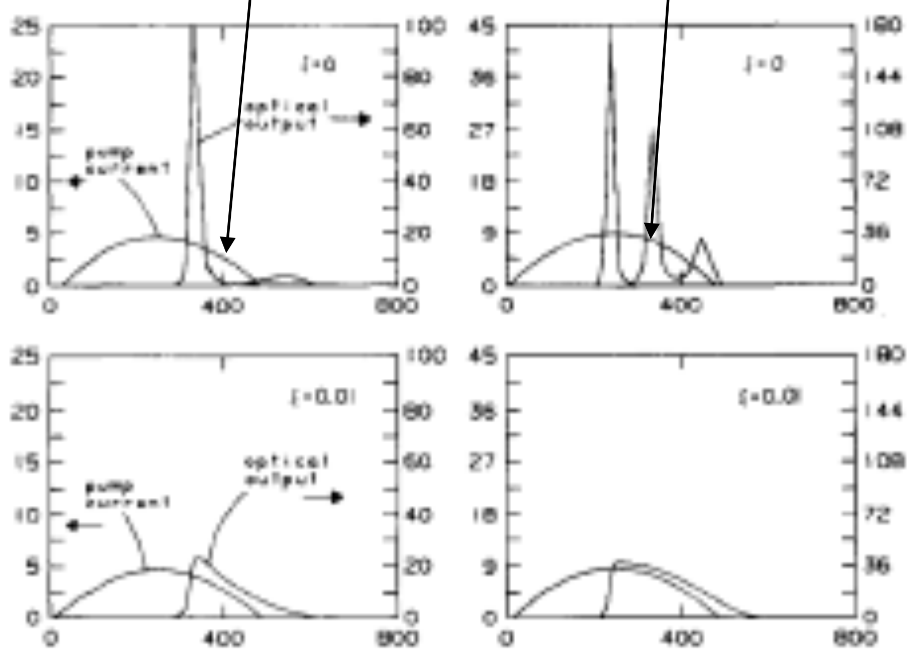
- So far we have studied lasers that emit continuous wave (CW) output power, and studied some aspects of small signal and large signal modulation dynamics.
- In this lecture, we will study a class of laser that emits pulses, or pulse trains, that can be further encoded with data using an external modulator. We will be studying modulators later in the class.
- The following types of lasers that emit pulses are of interest
  - Gain Switched: Directly turning on and off gain of the laser
  - Q-Switched: Periodically increasing the resonator loss (spoiling the Q) with an absorber in the laser cavity
  - Cavity Dumping: Storing photons in the resonator during the off-times and releasing the photons during the on-times.
  - Mode-Locked: Coupling and locking the phases of the cavity modes to each other.
- In this lecture we will talk briefly about gain switched and then concentrate on mode-locked lasers

Gain Switched Lasers

- Pulses are generated by rapidly modulating the laser gain via the injection current (usually with a sinusoid). The output pulse widths can be shorter than if limited by the carrier lifetime.
- Pulses with nanosecond to picosecond durations can be generated.
- The output pulse repetition rate can be adjusted by changing the frequency and bias of the current drive source.
- The laser is turned on from below threshold, and as we saw before with digital modulation, there will be feedback between the photon and carrier density, causing the laser to oscillate at the relaxation oscillation frequency until it is dampened.
- If the applied current pulse is turned off before the second ringing pulse appears, a very short pulse can be generated.
- In the figures at right, ξ is the gain compression factor which plays the role of dampening

Current pulse is turned off before second oscillation

Current pulse remains on longer than oscillation frequency

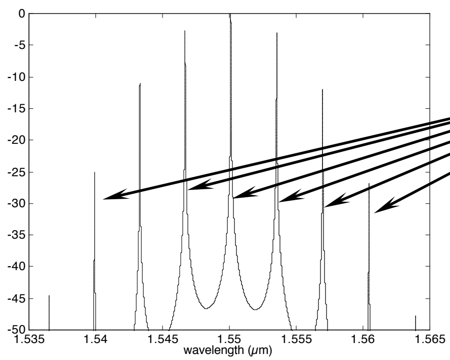


K. Lau, Journal Of Lightwave Technology, Vol. **7.** No. **2.** February **1989**o

Mode-locked Lasers

- Multi-mode lasers emit multiple frequencies that act as independent sources (i.e. the modes are not phase locked with each other). These frequencies are separated (in a FP laser) by the mode spacing $\Delta\nu = c/2nL$. Consider $M = 2S+1$ modes.
- If we use a means to lock the phase of these modes together, then the laser modes at as a single coherent source and we can apply the Fourier Transform to see that the laser will emit a periodic pulse train

Multi-Mode Laser



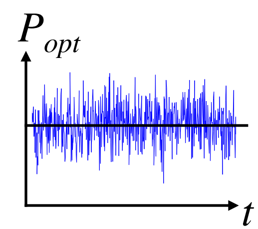
Independent oscillating modes

Mode amplitude

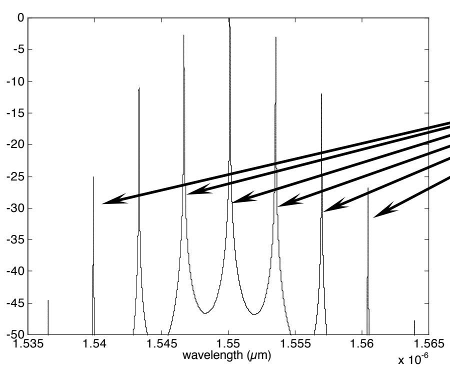
$$E_{out,total} = \sum_{i=-S}^{S} E_i \cos(\omega_i t + \phi_i)$$

Mode frequency

Independent random variable



Mode-Locked Laser



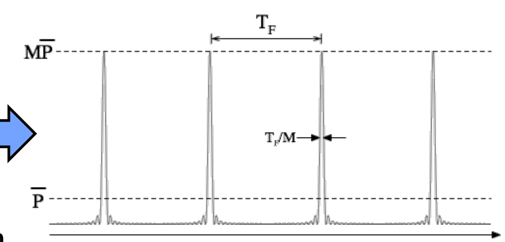
Phase-locked oscillating modes

Mode amplitude

$$E_{out,total} = \sum_{i=-S}^{S} E_i \cos(\omega_i t + \phi)$$

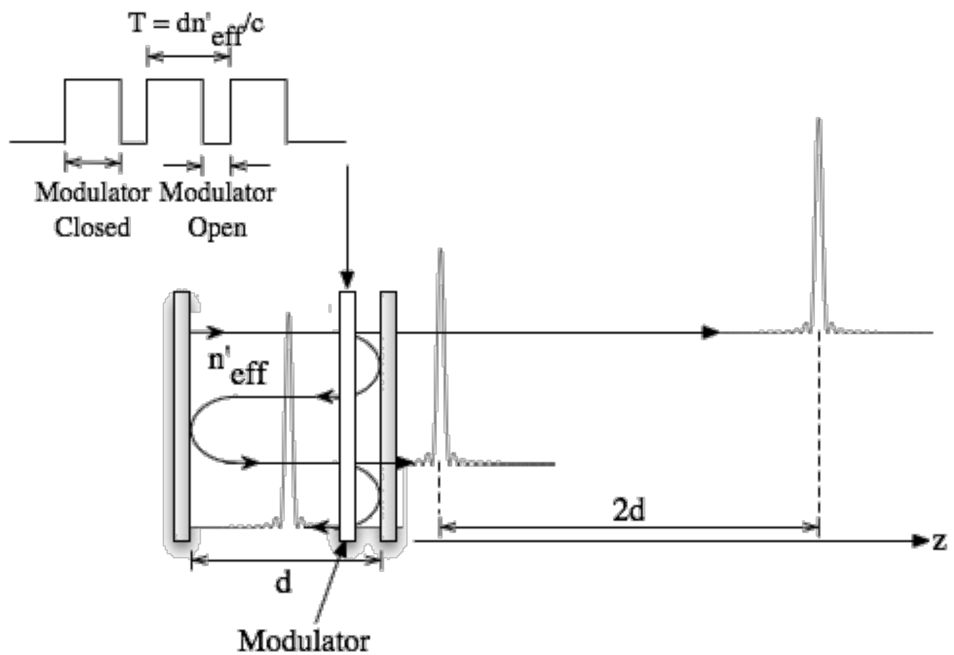
Mode frequency

Fixed phase for each mode



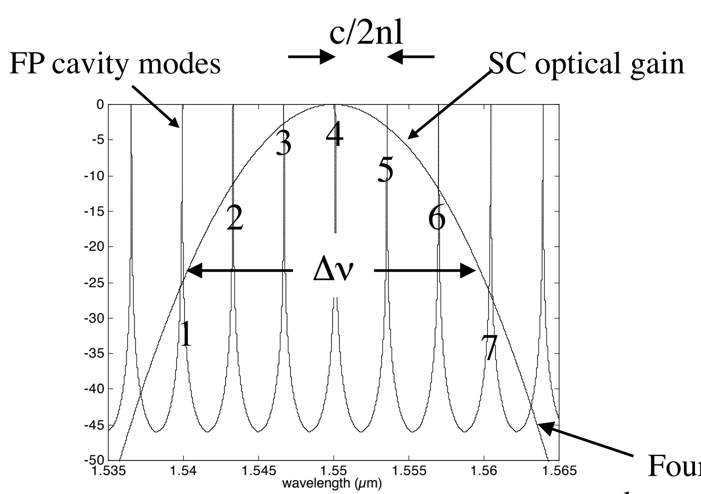
Mode Locked Lasers

- In order to lock the modes together; a loss element that is periodically modulated is placed inside the cavity. The modulater is driven at a frequency that creates a low loss condition only for pulses that match the cavity length.
- The cavity phase relationship will only be reinforced for this set of pulses, and energy that exits the cavity (in the form of these pulses) will be coherent with the cavity round trip time.
- Note that the modulator has to let pulses pass through going both directions in the cavity, hence it runs at twice the rate as the pulses exiting the cavity.

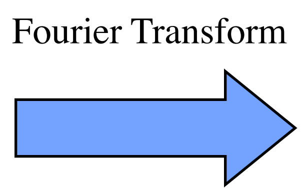


Mode Locked Lasers

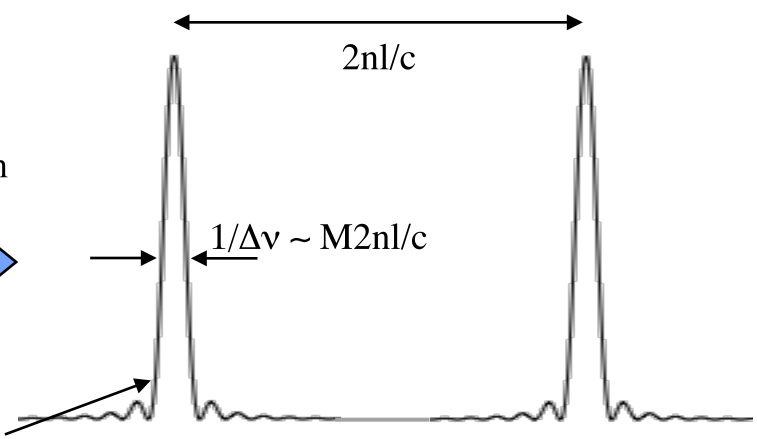
- Consider a laser that in steady state (non-modulated) contains M modes. Recall that the spacing between the modes is determined by the resonator (s) and the overall mode shape is determined by the material gain bandwidth as shown below. These can be treated as independent oscillators (incoherent w.r.t. each other).
- Once the modulator is turned on (at a rate equal to the cavity transit time), pulses are emitted from the cavity. The shape of these pulses is determined by the coherent superposition of phase locking the original modes together. Fourier Transform theory tell us the relationship between the frequency and time domain.



Laser modes in steady state operation



Fourier transform relation between gain shape and pulse envelope



FT of laser modes during mode-locking